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BT-I/D-20 **41046**

CALCULUS & LINEAR ALGEBRA

Paper : BS-133A

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. All questions carry equal marks.

UNIT-I

1. (a) Prove that $\Gamma m\,\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma 2m$ where Γ represents the gamma function and $\sqrt{\ }$ is the square root function.

(b) Find the volume of a sphere of radius a.

2. (a) Show that $\lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}}-e+\frac{ex}{2}}{x^2}=\frac{11e}{24}$.

(b) State Cauchy Mean value theorem and verify Cauchy Mean value theorem for the functions e^x and e^{-x} in the interval (a, b).

UNIT–II

3. (a) If $A=\begin{bmatrix}1 & 3 & 2\\ 2 & 0 & -1\\ 1 & 2 & 3\end{bmatrix}$, and I is identity matrix of order 3, evluate. $A^3-4A^2-3A+11I$.

(b) Solve the following system of equations using Cramer's rule.

$$x+3y+6z=2$$
$$3x-y+4z=9$$
$$x-4y+2z=7.$$

4. (a) Find the rank of the matrix

$$\begin{bmatrix}8 & 1 & 3 & 6\\ 0 & 3 & 2 & 2\\ -8 & -1 & -3 & 4\end{bmatrix}.$$

(b) Find the inverse of the matrix $\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$ and verify $A^{-1}.A$ = I where I is identity matrix of order 3.

UNIT–III

5. (a) For what value of k will the vector $u = (1, k, 5)$ in V_3 (R) be a linear combination of vectors $v = (1, -3, 2)$ and $w = (2, -1, 1)$.

(b) Show that the set {(2, 4, –3), (0, 1, 1), (0, 1, –1)} forms a basis of R^3.

6. (a) Show that the transformation $T : R^2 \to R^3$ defined by $T(x, y) = (x, x - y, x + y)$ is a linear transformation.

(b) For the linear transformation $T : R^2 \to R^3$ such that $T(x, y) = (x - y, y - x, -x)$, find a basis and dimension of its range space and its null space. Also verify, that $rank\ (T) + nullity\ (T) = \dim R^2$.

UNIT–IV

7. (a) Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

(b) Let V be an inner product space. Show that

$$\|u + v\| \leq \|u\| + \|v\|.$$

8. (a) Define Orthogonal Matrix; also show that the matrix

$$\begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

is orthogonal.

(b) If A is a square matric, show that (i) $A + A'$ is symmetric, (ii) $A - A'$ is skew-symmetric, where A' is the transpose of A'.